ACI – THE FINANCIAL MARKETS ASSOCIATION

EXAMINATION FORMULAE – 2009 VERSION

In all the formulae:

- interest rates, yields, coupon rates and rates of discount are expressed as a decimal, eg 8.53% will be expressed as 0.0853
- 'annual basis' is the number of days in a year assumed under the appropriate rate convention
- 'term' is the number of days from settlement to maturity of the instrument in question
- 'day count' is the number of days from settlement to maturity of the instrument in question.

INTEREST RATE CONVERSIONS

Converting between bond basis and money market basis (Act/360)

$$\text{rate}_{\text{bond basis}} = \text{rate}_{\text{money market basis}} \frac{365}{360}$$

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Converting between annually and semi-annually compounding frequencies

$$\text{rate}_{\text{annually-compounded}} = \left(1 + \frac{\text{rate}_{\text{semi-annually compounded}}}{2}\right)^2 - 1$$

$$\text{rate}_{\text{semi-annually compounded}} = \left(\sqrt{1 + \text{rate}_{\text{annually compounded}}} - 1\right)2$$

The formulae for converting between annually and semi-annually compounded rate apply only to rates quoted on a bond basis, not a money market basis.

MONEY MARKET

Certificates of deposit

$$\text{proceeds at maturity} = \text{face value}\left(1 + \frac{\text{coupon x term}}{\text{annual basis}}\right)$$

$$\text{secondary market proceeds} = \frac{\text{proceeds at maturity}}{1 + \frac{\text{yield x day count}}{\text{annual basis}}}$$

Discount-paying instruments quoted as a true yield

$$\text{secondary market proceeds} = \frac{\text{face value}}{1 + \frac{\text{yield x day count}}{\text{annual basis}}}$$

Discount-paying instruments quoted as a rate of discount

$$\text{discount amount} = \text{face value}\ \frac{\text{rate of discount x day count}}{\text{annual basis}}$$

$$\text{secondary market proceeds} = \text{face value}\left(1 - \frac{\text{rate of discount x day count}}{\text{annual basis}}\right)$$

$$\text{true yield} = \frac{\text{rate of discount}}{1 - \frac{\text{rate of discount x day count}}{\text{annual basis}}}$$

Forward price of sell/buy-back

$$\text{forward price} = \frac{(\text{repurchase price - accrued interest on collateral at termination})}{\text{nominal price of collateral}}100$$

FORWARD-FORWARDS & FORWARD RATE AGREEMENTS

$$\text{forward - forward rate} = \left[\frac{1+\dfrac{\text{interest rate}_{\text{long period}}\ \text{x day count}_{\text{long period}}}{\text{annual basis}}}{1+\dfrac{\text{interest rate}_{\text{short period}}\ \text{x day count}_{\text{short period}}}{\text{annual basis}}}-1\right]\frac{\text{annual basis}}{\text{day count}_{\text{forward-forward period}}}$$

$$\text{FRA settlement amount} = \text{notional principal amount}\ \frac{\left(\dfrac{(\text{FRA rate - settlement rate})\ \text{x day count}}{\text{annual basis}}\right)}{\left(1+\dfrac{\text{settlement rate x day count}}{\text{annual basis}}\right)}$$

FIXED INCOME

Clean and dirty price of bond with annual coupons on coupon date

$$\text{price} = 100\left[\left(\frac{\text{coupon}}{\text{yield}}\left(1-\frac{1}{(1+\text{yield})^{\text{remaining coupons}}}\right)\right)+\frac{1}{(1+\text{yield})^{\text{remaining coupons}}}\right]$$

Dirty price of bond with annual coupons

$$\text{dirty price} = \frac{\text{first cashflow}}{(1+\text{yield})^{\frac{\text{days to next coupon}}{\text{annual basis}}}}+\frac{\text{second cashflow}}{(1+\text{yield})^{1+\frac{\text{days to next coupon}}{\text{annual basis}}}}+\cdots+\frac{\text{n}^{\text{th}}\text{ cashflow}}{(1+\text{yield})^{(\text{n-1})+\frac{\text{days to next coupon}}{\text{annual basis}}}}$$

Duration at issue or on a coupon date

$$\text{Macaulay Duration} = \frac{\left[\begin{array}{l}(\text{present value of first coupon amount x time to first coupon})+\\ \quad(\text{present value of second coupon amount x time to second coupon})+\ldots\\ \quad\quad+(\text{present value of }(\text{last coupon amount}+\text{nominal amount})\text{ x time to last coupon})\end{array}\right]}{\text{net present value of bond}}$$

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{\left(1+\frac{\text{yield}}{\text{compounding frequency}}\right)}$$

Calculating zero-coupon yield from an annual yield-to-maturity (bootstrapping)

$$\text{zero-coupon yield for n-year term} = \left(\sqrt[n]{\frac{\text{final coupon amount} + \text{nominal amount}}{\text{implied present value of final coupon and nominal amount}}} - 1\right)100$$

The implied present value of the final coupon and nominal amount is calculated by subtracting from the net present value of the bond the sum of the present values of all coupons except the final one, where each present value is calculated using the appropriate zero-coupon yield.

FOREIGN EXCHANGE

Forward FX rate

$$\text{forward rate} = \text{spot rate}\ \frac{1 + \dfrac{\text{interest rate}_{\text{quoted currency}} \text{ x day count}}{\text{annual basis}_{\text{quoted currency}}}}{1 + \dfrac{\text{interest rate}_{\text{base currency}} \text{ x day count}}{\text{annual basis}_{\text{base currency}}}}$$

Covered interest arbitrage

synthetic quoted currency interest rate =

$$\left[\left(\left(1 + \frac{\text{interest rate}_{\text{base currency}} \text{ x day count}}{\text{annual basis}_{\text{base currency}}}\right)\frac{\text{forward rate}}{\text{spot rate}}\right) - 1\right]\frac{\text{annual basis}_{\text{quoted currency}}}{\text{day count}}$$

synthetic base currency interest rate =

$$\left[\left(\left(1 + \frac{\text{interest rate}_{\text{quoted currency}} \text{ x day count}}{\text{annual basis}_{\text{quoted currency}}}\right)\frac{\text{spot rate}}{\text{forward rate}}\right) - 1\right]\frac{\text{annual basis}_{\text{base currency}}}{\text{day count}}$$

OPTIONS

Standard deviation

$$\text{standard deviation} = \sqrt{\frac{\sum_{t=1}^{n}\left(\text{return at time t} - \text{mean return}\right)^2}{\text{number of observations} - 1}}$$

Calculating the volatility over a period from annualised volatility

$$\text{volatility over period t} = \text{annualised volatility}\sqrt{\text{t}}$$

Where t is in years or fractions thereof.

In standard deviation calculations the ACI exams assume a year of **252** working days.